Optimal Dynamic Risk-Taking

Ajay Subramanian  Baozhong Yang

Robinson College of Business
Georgia State University

African Institute of Mathematical Sciences

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Dynamic project selection/risk-taking problems arise naturally in financial economics

Asset substitution
- By choosing risky projects, shareholders can transfer wealth from debtholders and lower total firm value

Risk management
- How managers manage risk in response to their payoff structure

Financial regulation
- Control the risk-shifting incentives of managers or shareholders of financial institutions
Overview of Results I: The Model

- We analyze a continuous-time stochastic control problem
  - arises in the study of aforementioned issues
- An agent controls the evolution of a diffusion output process by
  - dynamically selecting one of an arbitrary (but finite) number of projects
  - also chooses the termination time.
- The agent has a general utility or payoff function
We analytically characterize the optimal policy.
Depends on the projects’ *risk-adjusted drifts*—drifts, volatilities and curvature of agent’s payoff function.
The optimal policy only selects among the *spanning projects*.
If projects’ risk-adjusted drifts are *consistently ordered*:
- with $K$ spanning projects, there are at most $K - 1$ switching points in the optimal policy.
Agent starts with a spanning project with the highest risk-adjusted drift when output is sufficiently high; as output decreases, progressively switches to a spanning project with *higher volatility* and *lower risk-adjusted drift*.
The switching triggers and termination threshold are analytically characterized.
Without consistent ordering condition—general and tractable algorithm to derive optimal policies.
Overview of Results III: An Application

- Analyze a structural model of capital structure
  - The firm can switch between two projects
  - Systematic risk can affect risk premium and thus the risk-neutral drifts of the projects
- Leverage ratios depend non-monotonically on systematic risk
- Substantial agency costs
- Leverage ratios and credit spreads consistent with empirical values
Broad class of one-dimensional optimal stochastic switching models

- drifts and volatilities of the underlying output process as well as the agent’s payoff functions may depend on different regimes (see e.g. Brekke and Øksendal (1994), Duckworth and Zervos (2001), Pham (2007), Pham and Vath (2007), Guo and Tomecek (2008), and Pham, Vath, and Zhou (2009))

- Pham (2007)—proofs of regularity and existence of solutions to certain one-dimensional optimal switching models (see also Bayrakhtar and Egami (2010))
  - difficult to obtain explicit characterizations of the solutions, even in the simplest case where there are only two projects or regimes.

- Pham and Vath (2007) provide explicit solutions in the special case of two projects with a power payoff function.

- Bayrakhtar and Egami (2010) provide sufficient conditions under which the solution assumes simple forms.
In the case of multiple projects, problem becomes even more difficult because the solution involves not only when to switch, but also which project to switch to.

Pham, Vath, and Zhou (2009) deduce qualitative properties of the optimal policy when the drift and volatility of the underlying process do not change, but the payoff functions can change across regimes.

They provide a characterization of the solution in a three-regime case under certain conditions on the payoff functions.

Our problem—related to general bandit problem—optimal policies unknown (see e.g., Gittins (1979), Banks and Sundaram (1992), and the survey in Mahajan and Teneketzis (2007))
Classical multi-armed bandit problem, the choice of an arm changes the state of that arm, *but leaves the states of all the other arms frozen.*

At any given time, only the state of the active arm can evolve and the arms follow independent processes.

“Gittins index” policy (Gittins (1979)) applies and the optimal policy can be characterized by indices that depend on each arm independently.

General bandit problem, if the states of the arms are not independent or can evolve simultaneously (“restless bandit problems”), index policies are no longer optimal and the general form of the optimal policy is unknown.

Heuristic algorithms are usually used in solving the general problem (see e.g., Mahajan and Teneketzis (2007)).

In our problem, all the projects depend on the output process that evolves all the time. Therefore, the Gittins-type index policy does not apply.
We consider general class of payoff functions and multiple projects/regimes that could differ in their drifts and volatilities.

Jointly analyze the optimal project selection and optimal termination problems.

- aforementioned papers preclude the possibility of termination by making nonnegativity assumptions on the agent’s payoff function.
- incorporation of the possibility of termination has a major impact on the nature of optimal policies.
- makes our model relevant to the study of asset substitution, risk management, and related problems, where the possibility of bankruptcy plays a central role (see Leland (1998)).

Relative to some of the above papers, however, we abstract away from switching costs.

In typical applications of our framework such as asset substitution and financial risk management, switching costs are relatively insignificant.
Our model also related to literature that studies investment models stemming from the seminal work of Merton (1971) on optimal investment strategies of a rational investor.


Cadenillas, Cvitanic, and Zapatero (2007) and Ou-Yang (2007) investigate models of delegated portfolio management where an agent makes dynamic investment decisions that affect the wealth process.

Dybvig and Liu (2011) analyze models of optimal consumption and investment with an optimally chosen retirement or termination date.

In these models, agent invests in any linear combination of a number of securities. Our model corresponds more closely to corporate investment or operational decisions—discrete set of projects available—impossible to “short” a project.
Model

- Continuous time horizon: \( t \in [0, \infty) \)
- All stochastic processes are defined on an underlying probability space \((\Omega, \mathcal{F}, \mathbb{P})\).
- A firm generates cash flows \( C_t \) that evolve as follows

\[
dC_t = \mu(P_t)C_t dt + \sigma(P_t)C_t dB_t
\]

- \( p_t \in \{1, 2, \ldots, N\} \) represents the project choice at date \( t \)

\[
\mu(P_t) = \mu_i, \quad \sigma(P_t) = \sigma_i, \quad \text{if } P_t = i,
\]

where \((\mu_i, \sigma_i)\) are constants for all \( 1 \leq i \leq N \)

- WLOG, assume

\[
\sigma_1 < \sigma_2 < \ldots < \sigma_N
\]
Admissible Policies

- Let \( \{ \mathcal{F}_t \} \) denote the complete and augmented filtration generated by the Brownian motion \( B \).
- \( P \equiv \{ P_t \} \) to be an admissible project choice process for the agent if it is an \( \{ \mathcal{F}_t \} \)–progressively measurable process that takes values in the finite set \( \{1, 2, \ldots, N\} \).
- There exists an increasing sequence of \( \{ \mathcal{F}_t \} \)–stopping times \( (\tau_0 = 0, \tau_1, \tau_2, \ldots) \) with \( \lim_{i \to \infty} \tau_i = \infty \) a.s. such that
  \[
P_t = P_{\tau_i}, \quad \tau_i \leq t < \tau_{i+1}.
  \]
- Let \( \mathcal{P} \) denote the set of admissible project choice processes.
- Let \( \tau \) be an \( \{ \mathcal{F}_t \} \)–stopping time that denotes the agent’s choice of the firm’s termination time.
The agent’s value from the policy $\Pi \equiv (P, \tau)$ is

$$V^\Pi(C_0) = E^P\left[\int_0^\tau e^{-rt}f(C_t)dt\right].$$  \hfill (1)

$E^P$ denotes the expectation under the probability distribution induced by the project choice process $P$, $f$ is the agent’s continuous payoff function and $r$ is the time discount rate.

$f$: the agent’s payoff function, strictly increasing, e.g.

- Shareholders: in the presence of debt and taxes, $f(C_s) = (1 - \tau)(C_s - \theta)$
- Manager: $f$ can be concave or convex depending on managerial risk aversion and the compensation structure
Objective

- We assume that
  \[ \mu_i < r, \quad 1 \leq i \leq N \]  
  \[ |f(x)| \leq c(1 + x), \quad \text{for } x > 0. \]  

- Above conditions ensure that the integral in (1) is finite.
- If \( f \) is concave (not necessarily strictly), the condition (3) is automatically satisfied.
- Even when \( f \) is convex, if the payoff function is asymptotically linear (for example, option compensation), condition holds.
- The objective of the agent is to solve
  \[ S(C_0) = \sup_{\Pi=(P,\tau)} V^\Pi(C_0). \]
Applications

- First, suppose the firm is controlled by its risk-neutral shareholders.
- The firm has a capital structure that consists of equity and long-term debt that requires it to make a total debt payment $\theta$ per unit time.
- In the absence of taxes, the payout flow to shareholders per unit time is
  \[ f(C_t) = C_t - \theta. \]
- In the above scenario, the optimal termination time is the time at which the shareholders optimally declare bankruptcy.
Asset Substitution

- In analyses of *asset substitution*, shareholders could choose risky, negative NPV projects that lead to a reduction in overall firm value.

- More specifically, consider the scenario (a special case of this scenario is studied by Leland (1998)) where shareholders can dynamically choose between two projects 1 and 2 where

\[ \sigma_1 < \sigma_2, \mu_1 > \mu_2. \]  

(5)

- The less risky project 1, therefore, has a higher NPV than the more risky project 2.

- The project that maximizes overall firm value is project 1.

- The dynamic project choice strategy or policy that maximizes *shareholder value*, however, could involve switching to the riskier project 2, especially when the firm is in financial distress.
Risk Management

- Shareholders of a leveraged firm may have lower incentives to choose safe, positive NPV projects because a large portion of the project’s cash flows may accrue to debtholders.

- In the context of our general framework, consider the case where the two projects satisfy

\[ \sigma_1 < \sigma_2, \quad \mu_1 < \mu_2. \]  

- In this scenario, the riskier project 2 has a higher NPV when the firm is in a normal state and, therefore, maximizes total firm value.

- In the presence of shareholder-debtholder agency conflicts, however, the incentives of shareholders could prevent them from using the less risky project 1 to hedge when the firm is in financial distress.
More generally, firm could have multiple projects with differing drifts and volatilities.

In addition, there can be nonlinear costs due to various sources such as taxes, financial distress costs and operating costs that affect shareholders’ payoffs.

Net payoff to shareholders $f(C_t)$ could differ in general from the earnings, $C_t$.

The analysis of shareholders’ optimal project choice policy and the shareholder-debtholder agency costs, therefore, entails solving the general stochastic control problem (4), where there are multiple available projects.
Managerial Risk-Taking

- Our framework is also applicable to the analysis of project choices by a manager whose objective function depends on her utility function and compensation structure and differs, in general, from that of shareholders.

- The manager’s optimal dynamic project choices, therefore, do not maximize shareholders’ value.

- In such cases, the analysis of the manager’s optimal project selection and the inefficiencies due to manager-shareholder agency conflicts requires solving the general stochastic control problem (4).

- Furthermore, the model can also be applied to the case of financial regulation where a regulator can impose restrictions on the firm’s project choices, in order to maximize a social objective function.
Suppose that there exists $C_B \geq 0$ and a function $S \in C^2(\mathbb{R}_+ \setminus C_B) \cap C^1(\mathbb{R}_+)$ with $S \geq 0$. Suppose that

$$\max_{i \in \{1, 2, ..., N\}} L_i S(C) + f(C) = 0 \text{ for } C > C_B,$$

(7)

$$S(C) = 0, \text{ for } C \leq C_B,$$

(8)

$$S(C) \leq c(1 + C),$$

(9)

where $c$ is a constant and

$$L_i S(C) = \frac{1}{2} \sigma_i^2 C^2 \frac{d^2 S}{dC^2} + \mu_i C \frac{dS}{dC} - rS, \quad i \in \{1, 2, ..., N\}.$$  

(10)

Then $S(C_0)$ is the value function that solves problem (4) when the initial cash flow is $C_0$. 

Define

\[ i^*(C) = \min \left( \arg \max_i L_i S(C) + f(C) \right) \]  

(11)

and the stopping time

\[ \tau_{CB} = \inf \left\{ t : C_t < C_B \right\}. \]  

(12)

Then \( i^*(C) \) is the optimal project choice when the cash flow is \( C \) and \( \tau_{CB} \) is the optimal termination time.
Markov Control Policies

- Given the Markov property of the cash flow process $C$, and the nature of the optimization problem (4), natural to consider Markov control policies in which the project choice only depends on the cash flow $C_t$.

- In particular, we will focus on policies with a finite number of switching points.

- For any set of $M$ points $Q = \{q_1, \ldots, q_M\} \subset \mathbb{R}_+$ such that $q_1 < q_2 < \ldots < q_M$, a sequence of projects $Y = (i_1, \ldots, i_{M+1})$ with $i_k \in \{1, \ldots, N\}, 1 \leq k \leq M + 1$, and a termination threshold $C_B < q_1$, we define the associated project selection policy $P = P(Q, Y)$ by

$$P_t = P(C_t) = i_k, \text{ for } q_{k-1} \leq C_t < q_k, \quad 1 \leq k \leq M + 1, \quad (13)$$

- The control policy $\Pi_{Q,Y,C_B} = (P(Q, Y), \tau_{C_B})$ is the switching policy with a finite number of switching points given by $Q$ and the termination time given by the $\tau_{C_B}$.

- We prove that, under very general conditions, the optimal control policy is a finite switching policy
Risk-Adjusted Drifts

- **Local curvature or relative risk aversion of** $f$ **at output level** $x$

$$
\gamma_f(x) = -\frac{xf''(x)}{f'(x)}
$$

- **Risk-adjusted drift** of project $i$ at $x$

$$
\tilde{\mu}_i(x) = \mu_i - \frac{1}{2} \gamma_f(x) \sigma_i^2
$$

- Risk-neutral shareholders: $\tilde{\mu}_i(x) = \mu_i$
- $f$ is CRRA: $\tilde{\mu}_i(x) = \mu_i - \frac{1}{2} \gamma \sigma_i^2$
Theorem

Assume \( f \in C^2 \), then

i) If \( \tilde{\mu}_1(x) > \tilde{\mu}_2(x) \) for all \( x > 0 \), then there exists a termination threshold level, \( C_B^* \geq 0 \), and a trigger \( \infty \geq q^* > C_B^* \) such that the agent's optimal project choice policy \( P^* = \{p_t^*\} \) is as follows:

\[
p^*_t = \begin{cases} 
1, & \text{if } C_t \geq q^*, \\
2, & \text{if } q^* > C_t \geq C_B^*.
\end{cases}
\]

ii) If \( \tilde{\mu}_1(x) \leq \tilde{\mu}_2(x) \) for all \( x > 0 \), then the agent always selects project 2.
Illustration of the Optimal Policy

The Agent Begins with Low−Risk Project 1

Switches to High−Risk Project 2

Switching Trigger $q^*$

Output Level: $C_t$

Time

Switches back to Low−Risk Project 1

Bankruptcy Threshold $C^*_B$

The Agent Declares Bankruptcy

The Model and Optimal Policy

Application: Systematic Risk and Capital Structure

Model

Two Projects

Multiple Projects

Conclusion
Intuition of the Optimal Policy

- Risk-adjusted drifts correspond to NPVs of the projects for the agent.
- When output level is high, the agent chooses the project with the higher NPV, or higher risk-adjusted drift.
- When output level is low and termination probability is high, the agent chooses the project with higher volatility (“option-like payoff”).
  - If project 1 has higher NPV, then the agent switches from low-risk project 1 to high-risk project 2.
  - If the high-risk project 2 also has higher NPV, then it dominates project 1.
- Proof of the *uniqueness* of the switching point uses the maximum principle for the elliptic Hamilton-Jacobi-Bellman equations.
Corollary

i) If $\mu_1 \geq \mu_2$ and $f$ is concave, then the optimal policy is characterized by a unique switching trigger, where the agent switches from the low-risk project 1 to the high-risk project 2 as output declines

ii) If $\mu_1 \leq \mu_2$ and $f$ is convex, then the agent always selects the high-risk project 2

Intuition: Concave/convex functions are limits of smooth functions
Two Projects: General Ordering

**Theorem**

Assume that the payoff function \( f \in C^2 \) and that there are

\[
0 = x_0 < x_1 < x_2 < \ldots < x_{M-1} < x_M = \infty
\]

such that for any \( 1 \leq i \leq M \), \( \tilde{\mu}_1(x) - \tilde{\mu}_2(x) \) does not change sign on \( x \in (x_{i-1}, x_i) \).

Then the total number of switching points in the optimal policy is bounded above by \( M \).

**Intuition:** In each region where the projects are “consistently ordered” by risk-adjusted drifts, the agent switches from the “higher” project to the “lower” project at most once.
Example: Optimal Policies with One Switching Point

\[ f(x) = \sqrt{x - 1}, \quad (\mu_1, \sigma_1) = (0.02, 0.1), \quad \sigma_2 = 0.2 \]
Example: Optimal Policies with Multiple Switching Points

- $(\mu_1, \sigma_1) = (-0.01, 0.1)$ and $(\mu_2, \sigma_2) = (0, 0.2)$
- Payoff function with a kink
  \[ f(x) = \begin{cases} 
  0.2x + 0.3, & \text{if } x \geq 1, \\
  x - 0.5, & \text{if } 0 \leq x < 1. 
  \end{cases} \]
- Optimal policy
  \[ p^*_t = \begin{cases} 
  2, & \text{if } C_t \geq 1.345, \\
  1, & \text{if } 1.345 > C_t \geq 0.921, \\
  2, & \text{if } 0.921 > C_t \geq 0.280. 
  \end{cases} \]

**Intuition:** it is optimal to hedge around the kink
Definition

A subset \( \{i_j\}_{1 \leq j \leq K} \) where \( 1 = i_1 < i_2 < \ldots < i_K = N \) is called a \textit{spanning subset} of the projects if

i) The drifts of projects in the subset are strictly concave in their variances,

\[
\mu_{i_k} > \frac{(\sigma^2_{i_l} - \sigma^2_{i_k})\mu_{i_j} + (\sigma^2_{i_k} - \sigma^2_{i_j})\mu_{i_l}}{\sigma^2_{i_l} - \sigma^2_{i_j}}, \quad \text{for} \ 1 \leq j < k < l \leq K. \tag{14}
\]

ii) For any \( i_k < j < i_{k+1} \), the drifts of the projects \( \{i_k, j, i_{k+1}\} \) are weakly convex in their variances,

\[
\mu_j \leq \frac{(\sigma^2_{i_{k+1}} - \sigma^2_{i_j})\mu_{i_k} + (\sigma^2_{i_j} - \sigma^2_{i_{k+1}})\mu_{i_{k+1}}}{\sigma^2_{i_{k+1}} - \sigma^2_{i_k}}. \tag{15}
\]
Intuition: Spanning Projects

The spanning subset is given by the *upper convex contour* of the points given by the variances and drifts of the projects.
Intuition: Spanning Projects

The spanning subset is given by the *upper convex contour* of the points given by the variances and drifts of the projects.
Theorem

Assume that \( f \in C^2(\mathbb{R}_+) \) and the agent’s optimal policy is characterized by a finite number of switching thresholds.

i) The agent only selects projects in the spanning subset \( \{i_j\}_{1 \leq j \leq K} \) in the optimal policy.

ii) At any switching point, the agent only switches between adjacent projects in the spanning subset.
Consistent Ordering of Projects

**Definition**

The risk-adjusted drifts of the spanning subset of projects are *consistently ordered* on an interval \((a, b)\) if there exists a permutation \(\{j_1, \ldots, j_K\}\) of the indices \(\{1, \ldots, K\}\) such that

\[
\tilde{\mu}_{ij_1}(x) \geq \tilde{\mu}_{ij_2}(x) \geq \cdots \geq \tilde{\mu}_{ij_K}(x), \quad \text{for all } a < x < b.
\]
Theorem

Assume that $f \in C^2(\mathbb{R}_+)$ and the risk-adjusted drifts of the spanning subset of projects are consistently ordered on $(0, \infty)$. Let project $i_h$ be the spanning project with the highest risk-adjusted drift. There exist a termination threshold level $C_B^* \geq 0$ and switching points $C_B^* = q_K^* < q_{K-1}^* < \ldots < q_h^* < q_{h-1}^* = \infty$ such that the agent’s progressively switches to spanning project with higher volatility:

$$p_t^* = i_k, \text{ if } q_k^* \leq C_t < q_{k-1}^*, \text{ for } h \leq k \leq K.$$
Intuition: Multiple Projects

Suppose project 2 has the highest risk-adjusted drift. Then the agent progressively selects project 2, 4 and 7.
General Ordering Case: Multiple Projects

**Theorem**

Assume that the payoff function $f \in C^2$ and that there are

$$0 = x_0 < x_1 < x_2 < \ldots < x_{M-1} < x_M = \infty$$

such that for any $1 \leq i \leq M$, the risk-adjusted drifts of the spanning subset of projects are consistently ranked on $(x_{i-1}, x_i)$. Then the total number of switching points in the optimal policy is bounded above by $M(K-1)$. 
Example: A Case with Three Projects

\[ f(x) = x - 1, \quad (\mu_1, \sigma_1) = (0.02, 0.1), \sigma_2 = 0.2, (\mu_3, \sigma_3) = (0, 0.3) \]

Critical value of \( \mu_2 \): \[ \bar{\mu}_2 = \frac{(\sigma_3^2 - \sigma_2^2)\mu_1 + (\sigma_2^2 - \sigma_1^2)\mu_3}{\sigma_3^2 - \sigma_1^2} = 0.0125 \]
Analytical Characterization and Computational Algorithm

- Step 1. Find the spanning subset of projects
- Step 2. Check the consistent ordering of projects
- Step 3. Closed form representation of the agent’s value function

\[
S(C) = \begin{cases} 
A_1 C^{\gamma_i_h} + g(C), & \text{if } C > q_h \\
A_2 C^{\gamma_{i_{h+1}}} + A_3 C^{\gamma_{i_{h+1}}} + g(C), & \text{if } q_{h+1} \leq C < q_h \\
\vdots \\
A_{2K-2h} C^{\gamma_{i_{K}}} + A_{2K-2h+1} C^{\gamma_{i_{K}}} + g(C), & \text{if } C_B \leq C < q_{K-1}
\end{cases}
\]

- Step 4: The switching thresholds and bankruptcy trigger are determined by the following $K - h$ “super-contact” conditions,

\[
S''(x)\big|_{x=q_j^+} = S''(x)\big|_{x=q_j^-} \quad j = h, \ldots, K - 1
\]

and the “smooth-pasting” condition $S'(C_B) = 0.$
A firm’s earnings evolve as follows

\[
\frac{dC_t}{C_t} = \hat{\mu}_{p_t} dt + \sigma_{p_t} dB_t, \quad p_t \in \{1, 2\}
\]

The drifts and volatilities of the projects satisfy

\[
\hat{\mu}_1 < \hat{\mu}_2 \quad \text{and} \quad \sigma_1 < \sigma_2
\]

The risk-neutral drifts satisfy

\[
\mu_i = \hat{\mu}_i - \lambda \sigma_i, \quad i = 1, 2.
\]

Market price of risk \(\lambda\) increases with the systematic portion of risk.
Application on Systematic Risk, Risk-Taking and Capital Structure II

- At time $t = 0$, the firm issues long-term debt with perpetual coupon $\theta$
- Cash flow to shareholders is $(1 - \tau)(C_t - \theta)dt$
- Cash flow to debtholders is $\theta dt$
- Shareholders dynamically select projects and determine bankruptcy time $\tau_B$
- Upon bankruptcy, the salvation value is $(1 - \alpha)$ times the unlevered firm value
The risk neutral drifts satisfy

$$\mu_i = \hat{\mu}_i - \lambda \sigma_i, = \bar{\mu} + (\beta - \lambda)\sigma_i, \quad i = 1, 2.$$ 

The optimal policy has a **structural change** at $\lambda = \beta$, where the order of the risk-neutral drifts changes.
We calibrate parameters using market data and corporate data from Compustat large non-financial, non-utility firms over the period 1962 – 2009.

<table>
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<th>Description</th>
<th>Parameter</th>
<th>Value</th>
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<td>Bankruptcy cost</td>
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Analyze a general class of dynamic risk-taking models applicable to financial economics
First provide analytical characterizations of optimal policies

- Optimal policies depend on risk-adjusted drifts of projects
- Only projects in the spanning subset are selected
- If there are \( K \) spanning projects with consistently ordered risk-adjusted drifts, the optimal policy is characterized by at most \( K - 1 \) unique switching triggers
- The agent progressively switches from projects with higher risk-adjusted drifts and lower volatility to those with lower risk-adjusted drifts and higher volatility
Conclusion II

Concrete application to capital structure with asset substitution

- Optimal policy, leverage, and agency costs depend on **systematic risk**
- There can be **substantial agency costs**
- Generate leverage ratios and credit spreads consistent with empirical values