Tutorial: Structural Models of the Firm

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Tutorial: Structural Models of the Firm
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Structural Models of the Firm

- We model the dynamics of the balance sheet of a firm and establish conditions under which the assets of the firm cannot support the debt.
- Since these models explicitly model the behavior of the assets of the firm they are often referred to as Firm Value models.
- Default occurs if the asset value is not sufficient to cover the firm’s liabilities.
- The distinguishing feature of structural models is that they view equity and debt as contingent claims on the assets of the firm.
- Given the value of the assets of the firm, together with how the asset value evolves over time, and given the capital structure of the firm, the primary goal is to identify probabilities of the firm defaulting over given time horizons.
The firm’s value process is not observable, so we need to infer the dynamics from observable prices of the claims that depend on the asset value.

These claims include equity prices, option prices on the equity, debt prices, and credit default swap rates.

The models may require information on dividends, recovery rates by industry and possibly by seniority, riskless yield curve information and other market related information.

Taxes, Bankruptcy rules, equity issuance costs, and other market frictions can enter the analysis.

The models differ in how precisely they account for the specific capital structure of the firm. They also differ in how they model bankruptcy itself.
Example (From KMVs web site)

Credit Monitor

Market Value of Assets

Defaulterd
November 2001

Default Point
(Liabilities Due)
# Structural Models of the Firm

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**Structural Model**

- Distance to Default
- Probability of Default
- Risk Neutral Probability of Default
- Survival Probabilities
- Term Structure of Credit Spreads
- Asset Value and Asset Volatility
- Impact on Spreads with Changing Capital Structure

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*Peter Ritchken, Case Western Reserve University*
Consider a corporation that has a capital structure consisting of equity and a single issue of discount bonds of total face value $F$ that is due at time $T$. Let $B_0$ denote the current value of the bond issue and let $E_0$ be the current total value of outstanding shares. The current value of the firm is $V_0 = B_0 + E_0$. If the firm cannot pay the face value at maturity, then the firm defaults.
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Assume the bond indenture prohibits dividend payments during the life of the bond and also prevents the firm from issuing senior debt, repurchasing shares, or selling assets. The current value of the firm is

$$V_0 = B_0 + E_0$$
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The Merton Model of the Firm

- The value of the assets of a firm follow a Geometric Wiener process of the form

\[ \frac{dV_t}{V_t} = \mu_V dt + \sigma_V d\omega_t \quad V_0 \text{ given} \]
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- The logarithmic return over the time period \([0, T]\) is:
  \[ r_T = \alpha_V T + \sigma_V \sqrt{T} Z_T \]

  where \( \alpha_V = \mu_V - \sigma_V^2 / 2 \).
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- If we knew \(V_0\), the face value \(F\), \(\alpha_V\) and \(\sigma_V\), we could easily determine the probability of default:
  \[
  P(\text{Default}) = P(V_T < F) = P(\ln(V_T/V_0) < \ln(F/V_0))
  = P(r_T < \ln(F/V_0))
  = P \left( \alpha_V T + \sigma_V \sqrt{T} Z < \ln(F/V_0) \right)
  = P \left( Z < \frac{\ln(F/V_0) - \alpha_V T}{\sigma_V \sqrt{T}} \right)
  = N \left( \frac{\ln(F/V_0) - \alpha_V T}{\sigma_V \sqrt{T}} \right)
  \]
The distance to default, DD, measures how far the expected logarithmic return $\alpha_v T$ is from the default boundary $\ln (F/V_0)$ in units of standard deviations. Specifically we have

$$DD = \frac{\alpha_v T - \ln(F/V_0)}{\sigma \sqrt{T}}$$
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The greater this distance the greater the cushion of assets that support the liabilities. The probability of default can then be written as:

$$P(\text{Default}) = N(-DD)$$
EDF: Expected Default Frequency:

- Distribution of asset value at horizon
- Asset Volatility (1 Std Dev)
- Distance-to-Default = 3 Standard deviations
- Default Point
- EDF

Value
Asset Value
Distance-to-Default
Today 1 Yr Time
Example: Merton Model of the Firm

- Assume a firm has asset value, $V_0 = 100$, and asset volatility, $\sigma_V = 0.30$. The firm has outstanding one year ($T = 1$) debt with a face value of $F = 60$. The risk free rate for a year is 5% continuously compounded. The firm’s asset beta value is estimated as $\beta = 1.5$, and the market risk premium is taken as 6%.

\[ \mu_V = r_f + \beta (\mu_M - r_f) = 0.05 + 1.5(0.06) = 0.14 \]

Once $\mu_V$ is given, we can compute $\alpha_V = \mu_V - \sigma^2_V/2 = 0.14 - 0.30^2/2 = 0.095$.

We now can compute the default probability over the one year time horizon. The distance to default is given by

\[ DD = \alpha_V T - \ln\left(\frac{F}{V_0}\right) \sigma_V \sqrt{T} = 0.095 - \ln\left(\frac{60}{100}\right) \times 0.30 = 2.0194 \]

The probability of default is therefore given by

\[ P(\text{Default}) = N(-2.0194) = 2.17\% \]
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- Using the capital asset pricing model, the expected growth rate of the assets of the firm is computed as:

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Pricing Equity with The Merton Model

- If at date $T$, the firm value exceeds the face value of debt, then the firm pays its obligations, $F$, and the equity holders are left with the residual; otherwise the firm defaults and the equity holders get nothing.

- The equityholders payout is

$$E_T = \text{Max}(0, V_T - F)$$

- The equityholders have a call option on the assets of the firm.

$$E_0 = V_0 N(d_{10}) - Fe^{-rfT} N(d_{20})$$

(1)

$$d_{10} = \frac{\ln(V_0/F) + (rf + \frac{1}{2}\sigma^2)T}{\sigma V \sqrt{T}}$$

$$d_{20} = d_{10} - \sigma V \sqrt{T}$$
Pricing Debt with The Merton Model

- The bondholders now own the firm but have sold the call option to the original owners.

That is

\[ B_T = \text{Min}(F, V_T) = V_T - \text{Max}(V_T - F, 0) \]
\[ = V_T - E_T \]

- The date 0 value of the claim is given by \( B_0 = V_0 - E_0 \)

- Now, the yield to maturity is \( y(0, T) \), where \( B_0 = Fe^{-y(0, T) T} \).

- This yield to maturity can be broken up into a riskless yield to maturity and a credit spread. That is \( y(0, T) = r_f + s(0, T) \)

- Substituting in for the equity price and rearranging

\[ s(0, T) = - \left( \frac{\ln\left(\frac{N(-d_{10})/L_0) + N(d_{20})}{T}\right)}{T} \right) \]

where \( L_0 = \frac{Fe^{-r_f T}}{V_0} \).
Pricing Debt with The Merton Model

- An alternative way of representing the value of the bond at maturity is:

\[ B_T = \text{Min}(F, V_T) \]
\[ = F - \text{Max}(F - V_T, 0) \]
\[ = F - P_T \]

where \( P_T \) is the value of a put option on the firm with strike price equal to \( F \).

- Hence, it must follow that

\[ B_0 = F e^{-r_f T} - P_0. \]

- Within this framework, shareholders can be viewed as owners of the assets of the firm who have borrowed the present value of \( F \) and purchased a put.

- The loan is an obligation that must be met, regardless of what occurs.

- Without the put option, the shareholders would not have limited liability. That is, at maturity, if the value of the firm was lower than \( F \), the shareholders would be obliged to pay the difference.

- The likelihood of defaulting is exactly equal to the likelihood of exercising the put.
The risk neutral probability of default is

\[
Prob_{RN}(\text{Default}) = P(V_0 e^{(r_f - \sigma_V^2/2)T + \sigma_V \sqrt{T} Z} < F)
\]

\[
= P(Z < \frac{\ln(F/V_0) - (r_f - \frac{1}{2} \sigma_V^2) T}{\sigma_V \sqrt{T}})
\]

\[
= N(-DD_{RN})
\]

and \(DD_{RN} = -d_{20}\).

The risk neutral probability of default is linked to the actual probability of default. Reconsider the distance to default:

\[
DD = \frac{\alpha_V T - \ln(F/V_0)}{\sigma_V \sqrt{T}}
\]

\[
DD_{RN} = \frac{(r_f - \sigma_V^2/2) T - \ln(F/V_0)}{\sigma_V \sqrt{T}}
\]

Hence:

\[
DD = DD_{RN} + \frac{\alpha_V + \sigma_V^2/2 - r_f}{\sigma_V \sqrt{T}} T
\]

\[
= DD_{RN} + \frac{\mu_V - r_f}{\sigma_V} \sqrt{T}
\]
ABZ is a privately held company, about to go public. It has no traded equity or debt.

Its assets consist of traded securities valued at $100m dollars.

The capital structure of the firm consists of two tranches: A five year maturity zero coupon bond with face value $60m, the second consisting of equity. The goal is to value the equity and debt tranches.

The volatility of the traded assets of the firm is given as $\sigma_V = 0.30, V_0 = 100, F = 60, T= 5, \sigma_V = 0.30, r_f = 0.05$. 

Therefore the equity is priced using the Black Scholes equation at $56.10$. Therefore the debt is $43.90$. The continuously compounded yield over 5-years is $-\ln(43.90/60)/5 = 6.25\%$. The credit spread of the debt is therefore 125 basis points.

If the firm increases its use of debt then the credit spread will widen. For example if $F = 80$, the equity will be $E_0 = 44.96$, the bond price will be $B_0 = 55.04$, and the yield to maturity will be $-\ln(55.04/80)/5 = 7.48\%$, for a credit spread of 248 basis points.
ABZ is a privately held company, about to go public. It has no traded equity or debt.

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Credit Spread Determination

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\[ E = BS(V_0, \sigma_V; F, T, r_f). \]
Implementing Merton

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- The volatility of the equity is given by Black and Scholes as 
  \[ \sigma_E = \sigma_V \frac{V_0}{E_0} \frac{\partial E_0}{\partial V} = \sigma_V \frac{V_0}{E_0} N(d_{10}) \]
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- If we know the equity price and equity volatility, then we have two equations in two unknowns.
1. Capital Structure Simplification

Transform the debt structure of the firm into a zero coupon bond with maturity $T$ and a face value, $F$. One common method is to choose the face value so that:

Face Value = All Debt Due in One Year + 0.5 Long Term Debt

Estimation of Parameters

While the equity price is observable, the equity volatility is not, and it typically has to be estimated from historical data. Since equity is an option on the assets of the firm, equity volatility is not a constant, and estimating it using simple sample statistics from historical data is therefore inappropriate.
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2. Estimation of Parameters
Implementing Merton

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   - The KMV Approach and the Maximum Likelihood Approach of Duan
KMV Approach for Asset Value and Asset Volatility

- Let \( \{E_t\}_{t=0}^n \) represent the time series of equally spaced equity prices. Let \( E_t = BS(V_t; \sigma_V) \) represent the equity pricing equation.
- Express the asset value as a function of the equity price: \( V_t = BS^{-1}(E_t; \sigma_V) \).
- Given the equity price, \( E_t \), the face value of the debt, \( F \), the time to expiration, the riskless rate, \( r_f \), and the volatility, \( \sigma_V \), we can compute the asset value, \( V_t \).
- The resulting time series of implied asset prices is given by \( \{\hat{V}_t(\sigma_V)\}_{t=0}^n \).
- We begin with a guess for \( \sigma_V \). Then, in the first phase, we use the observed equity prices, together with \( \sigma_V \) to estimate the time series of asset values, \( \{\hat{V}_t(\sigma_V)\}_{t=0}^n \).
- For the second phase we use the time series of implied asset values, \( \{\hat{V}_t(\sigma_V)\}_{t=0}^n \) to compute the time series of continuously compounded asset returns, \( \{\hat{r}_t(\sigma_V)\}_{t=1}^n \), where \( r_{t+1} = \ln(\hat{V}_{t+1}/\hat{V}_t) \).
- The sample variance of these numbers is then used to compute a new estimate for the variance of the asset returns, \( \sigma_V^2 \). This estimate is then used in the first phase again.
- After iterating several times, the two phase procedure eventually converges.
- The final output consists of a time series of asset values and an asset volatility.
- The most recent asset value is taken to be \( V_0 \) and the asset volatility is taken to be the volatility produced at the final iteration.
The Black Cox Model

- The original Merton model does not allow for a default before maturity.
- Bond indentures often contain safety covenants that allows bondholders to push a firm into bankruptcy between payment dates if certain covenant rules are broken.
  - Example: covenant could forbid a firm to exceed a certain debt to equity ratio. Breeching this requirement could allow the bondholders to demand their principal back.
- Black and Cox considered models where the time of default is given as the first passage time of the value process $V$ to a deterministic or random barrier.
- The challenge here is to appropriately specify the lower threshold $V_B$, and the recovery process given default
The Black Cox Model

- Assume the asset value of the firm follows a lognormal process with:
  \[ V_T = V_0 e^{\alpha V T + \sigma V \sqrt{T} Z} \]

- Assume the barrier grows at the same growth rate. Let \( B_0 \) be the initial barrier. Then at any time \( s \):
  \[ B_s = B_0 e^{(\alpha V + \frac{1}{2} \sigma^2 V) s} \]

- This assumption implies that as the firm grows, on average it increases its debt so that the debt has the same drift as the drift of the asset.

- Default does not occur as long as \( V_s > B_s \), for times \( s \), \( 0 < s < t \). Equivalently for all times \( s \), we require
  \[ V_0 e^{\alpha V s + \sigma V \sqrt{s} Z_s} > B_0 e^{(\alpha V + \frac{1}{2} \sigma^2 V) s} \]
  \[ -\frac{1}{2} \sigma^2 V s + \sigma V \sqrt{s} Z_s > \ln(B_0/V_0) \]

- Black and Cox show that the probability of the firm surviving up to date \( t \), \( PS(t) \) is:
  \[ PS(t) = N \left( \frac{-\frac{1}{2} \sigma^2 V t + \ln(V_0/B_0)}{\sigma V \sqrt{t}} \right) - \frac{V_0}{B_0} N \left( \frac{-\frac{1}{2} \sigma^2 V t - \ln(V_0/B_0)}{\sigma V \sqrt{t}} \right) \]
The Black Cox Bond Pricing Model

If the default barrier has the form $B_t = C e^{-\gamma(T-t)}$, then

$$B_0 = F e^{-rf T} N(z_1) - F e^{-rf T} y^{2v_1-2} N(z_2) + V_0 e^{-q T} N(z_3) + V_0 e^{-q T} y^{2v_1} N(z_4) + V_0 y^{v_1+v_2} N(z_5) + V_0 y^{v_1-v_2} N(z_6) - V_0 e^{-q T} y^{v_1+v_3} N(z_7) - V_0 e^{-q T} y^{v_1-v_3} N(z_8)$$

where

$$y = Ce^{-T}/V_0; \quad v_1 = (r_f - q - \gamma + \sigma_V^2/2)/\sigma_V^2$$

$$v_2 = \sqrt{\delta/\sigma_V^2}; \quad v_3 = \sqrt{\delta - 2\sigma_V^2 a/\sigma_V^2}$$

$$\delta = (r_f - q - \gamma - \sigma_V^2/2)^2 + 2\sigma_V^2 (r_f - \gamma)$$

$$z_1 = (\ln(V_0/F) + (r_f - q - \sigma_V^2/2) T)/\sigma_V \sqrt{T}$$

$$z_2 = (\ln(V_0/F) + 2 \ln(y) + (r_f - q - \sigma_V^2/2) T)/\sigma_V \sqrt{T}$$

$$z_3 = (- \ln(V/F) - (r_f - q - \sigma_V^2/2) T)/\sigma_V \sqrt{T}$$

$$z_4 = (\ln(V/F) + 2 \ln(y) + (r_f - q + \sigma_V^2/2) T)/\sigma_V \sqrt{T}$$

$$z_5 = (\ln(y) + v_2 \sigma_V^2 T)/\sigma_V \sqrt{T}$$

$$z_6 = (\ln(y) - v_2 \sigma_V^2 T)/\sigma_V \sqrt{T}$$

$$z_7 = (\ln(y) + v_3 \sigma_V^2 T)/\sigma_V \sqrt{T}$$

$$z_8 = (\ln(y) - v_3 \sigma_V^2 T)/\sigma_V \sqrt{T}$$
Example Assume the asset value is $V_0 = $100, the face value, $F = $70. The riskless rate is $r_f = 0.05$; the volatility is $\sigma_V = 0.25$; the time to maturity is $T = 5$ and $q = 0$. Further, assume $C = 60$ with $\gamma = 0$. So the default barrier is flat at 60 until the maturity date $T$.

Figure 4: Default probabilities Over Time
The value of the equity of a firm is $40.
The Black Cox Bond Example

- The value of the equity of a firm is $40.
- The firm has a $T = 5$ year zero coupon bond with face value $70$. 

Using solver in Excel, we can imply out the value of the assets of the firm and the volatility of the assets of the firm. For these case parameters we find $\sigma_V = 0.183$ and $V_0 = 93.48$. The fair price of the debt is then computed from the Black Cox model to be $53.48$ which yields a credit spread of $53.48$ basis points.
The Black Cox Bond Example

- The value of the equity of a firm is $40.
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- We assume the default boundary is $C = 54$ with $\gamma = 0$. 

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The Delianedis-Geske Model

- Delianedis and Geske account for more complex capital structures by creating two tranches of risky debt.
- They designate current liabilities and debt due in one year as short term liabilities, $F_1$ and total liabilities minus short term liabilities as long term liabilities, $F_2$.
- They then compute the duration of short term liabilities, $T_1$ say and the long term liabilities, $T_2$ say.
- At date $T_1$ the firm is obliged to make the payment of $F_1$.
- The firm cannot sell its assets to meet its obligation. Rather, the firm must go to capital markets and raise funds (equity or new debt) to finance the payments.
- Clearly, the ability to raise funds will depend on the amount of debt outstanding.
Delianides Geske Model

![Graph showing relationships between variables F₁, F₂, T₁, T₂, and V']

- F₁ and F₂ represent different states or variables.
- T₁ and T₂ indicate time periods or points in the model.
- V' is a variable that changes over time, impacting the other variables.
The Delianedis-Geske Model

- If the present value of all debt outstanding, together with the required payment, $F_1$, exceeds the value of the firm, then the shareholders will declare the firm bankrupt.

- Let $B_2(T_1)$ be the date $T_1$ value of the debt due at time $T_2$. Then the firm is solvent if:

$$V(T_1) > F_1 + B_2(T_1)$$

- If the firm is solvent, then refinancing is assumed to take place with equity.

- If the above condition is not met, then the firm is insolvent and bankruptcy occurs.

- Hence, there is a critical firm value, $V^* = V^*(T_1)$ above which the firm remains solvent.

- If the payment is made, then $B_2(T_1)$ can be computed using the Merton model.

$$B_2(T_1) = V(T_1) - C(V(T_1), F_2, T_2 - T_1, r_f, \sigma_V)$$

where $C()$ is the value of a call option with strike $F_2$, time to expiry, $T_2 - T_1$, risk free rate $r_f$, and underlying asset volatility, $\sigma_V$. 
The Delianedis-Geske Model

- Define the point $V^*$ to be the value of the firm at date $T_1$ which equates the value of the outstanding debt, $F_1$, together with the fair value of the debt due at time $T_2$.
- That is at the point $V^*$ we have:

$$V^* = F_1 + B_2(T_1)$$

$$= F_1 + (V^* - C(V^*, F_2, T_2 - T_1, r_f, \sigma_V))$$

- Hence, the value $V^*$ is the point at which:

$$C(V^*, F_2, T_2 - T_1, r_f, \sigma_V) = F_1$$

- Viewed from time 0, equity holders have a compound option on the assets of the firm.
- If all goes well, then at time $T_1$ they can exercise their claim, make the payment of $F_1$ dollars and receive a call option on the assets of the firm.
- Hence, at date 0, they have an option on an option, or a compound option. The strike price of the compound option is $F_1$. 
Geske shows that the initial value of the compound option is given by

\[ CO_0 = V_0 e^{-dT_2} M(a_1, b_1, \rho) - F_2 e^{-rfT_2} M(a_2, b_2, \rho) - e^{-rfT_1} F_1 N(a_2) \]

where

\[
\begin{align*}
a_1 &= \frac{\ln(V_0/V^*) + (r_f - d + \sigma^2_V/2)T_1}{\sigma_V \sqrt{T_1}} \\
a_2 &= a_1 - \sigma \sqrt{T_1} \\
b_1 &= \frac{\ln(V_0/F_2) + (r_f - d + \sigma^2_V/2)T_2}{\sigma_V \sqrt{T_2}} \\
b_2 &= b_1 - \sigma_V \sqrt{T_2} \\
\rho &= \sqrt{\frac{T_1}{T_2}}
\end{align*}
\]

and \( M(x, y, \rho) = P(X < x, Y < y) \) where \( X \) and \( Y \) are bivariate standard normal random variables with correlation, \( \rho \).
If the value of the firm and the volatility of the assets of the firm were available then we could use this model directly.

These numbers are not observable and so must be extracted from other sources. Use equity price and equity volatility, where:

\[
\sigma_E = \frac{\partial E}{\partial V} \frac{V}{E} \sigma_V = e^{-d_{T_1}} M(a_1, b_1, \rho) \frac{V}{E} \sigma_V
\]

This model allows risk neutral probabilities of default to be computed over different time horizons.

Total Default Probability = \(1 - M(a_2, b_2, \rho)\)

Short Term Default probability = \(1 - N(a_2)\)

Forward or long term conditional Default probability = \(1 - \frac{M(a_2, b_2, \rho)}{N(a_2)}\)
Binomial Lattice Models

- Take all obligations of the firm and map them into cash flows for each period corresponding to the periods of the binomial lattice.
- Assume a given initial asset value and asset volatility and construct a binomial lattice of asset values.
- Use backward recursion to value all the outstanding debt at each node, treating all the liabilities as one tranche of senior debt.
- If the value of the assets fall below the value of all outstanding collective debt, then the firm defaults, and at this node, the value of the equity is zero. Ultimately, the lattice of equity prices is obtained.
Since equity is a compound option, the volatility of equity is given by

\[ \sigma_E = \frac{\partial E}{\partial V} \frac{V}{E} \sigma_V \]

So to obtain the volatility of equity we perturb the initial value of the asset and compute the new equity value and then use the above equation.

If the value of the equity and the value of the volatility of equity match their input values, then our lattice is well calibrated.

At each node of the lattice we know the equity value, the total value of all outstanding liabilities and we also know if the firm defaults.

Many ways to calibrate the model.

Useful for pricing options on equity for highly levered firms.
In the above analysis we took the equity price as a given as well as the volatility of equity.

Rather consider dropping the requirement of matching equity volatility and instead calibrate an asset lattice using option information.
Models that use Option Information

- In the Merton model, equity is a call on the assets.
- Traded options on equity are options on options. These options can therefore be priced using the Geske compound option option model.
- If we select the right asset value and asset volatility, we should be able to match the equity price and the set of option prices.
The value of the firm is $V = 100$. The volatility of the assets is $\sigma_V = 0.2$. The riskless rate is 5%.
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A firm has a zero coupon bond with face value $F = 75$ and maturity $T = 2$ years. The equity of the firm is priced at its Black Scholes value of 33.04. The implied volatility of the equity is computed as 0.566.
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However, since the equity is an option on the assets of the firm, then options on equity should be priced by the Geske model.
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However, since the equity is an option on the assets of the firm, then options on equity should be priced by the Geske model.

Once the prices of options are computed, then the Black-Scholes model can be used to obtain their implied volatilities.
Assume the firm’s stock pays no dividends. The price of the stock is $10. Assume a 6 month at the money call option trades at $2.18. The firm has a five year zero coupon bond with face value $40 on a per share basis. The riskless rate is 5%.
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Guess $V_0$ and $\sigma_V$. Price Equity using the Black Scholes equation. Price options using the Geske model.

Use solver in excel to find the two values that equate the actual market values to their theoretical prices. For this example, the solution is $V = 32.31$ and $\sigma_V = 0.341$.

With these values the bond price can be computed as $B = V - E = 22.31$. An otherwise equivalent riskless bond would be priced at $31.15$. The yield to maturity of this bond is 11.68%, so the credit spread is about 668 basis points. We could calibrate the model to produce the market cds rate instead.
Models that use Option Information

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We could calibrate the model to produce the market cds rate instead.
Consider a firm that has issued a zero coupon bond with 1 year to maturity and a face value of $F = 60$. The equity price is 29.73.
Models that use Option Information

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The riskless rate is 1% and the dividend yield is also 1%. The strike, time to expiry, market price and option implied volatility are shown in the first four columns.

<table>
<thead>
<tr>
<th>Option Strike</th>
<th>Option Maturity</th>
<th>Option Price</th>
<th>Option IV</th>
<th>Compound Price</th>
<th>Compound IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.00</td>
<td>0.11</td>
<td>5.04</td>
<td>0.49</td>
<td>4.91</td>
<td>0.41</td>
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<tr>
<td>30.00</td>
<td>0.11</td>
<td>1.12</td>
<td>0.32</td>
<td>1.35</td>
<td>0.38</td>
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<tr>
<td>35.00</td>
<td>0.11</td>
<td>0.10</td>
<td>0.33</td>
<td>0.14</td>
<td>0.35</td>
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<tr>
<td>25.00</td>
<td>0.36</td>
<td>6.45</td>
<td>0.56</td>
<td>5.67</td>
<td>0.41</td>
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<tr>
<td>30.00</td>
<td>0.36</td>
<td>2.52</td>
<td>0.37</td>
<td>2.57</td>
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<tr>
<td>35.00</td>
<td>0.36</td>
<td>0.76</td>
<td>0.34</td>
<td>0.89</td>
<td>0.36</td>
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<tr>
<td>40.00</td>
<td>0.36</td>
<td>0.19</td>
<td>0.33</td>
<td>0.23</td>
<td>0.34</td>
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<tr>
<td>25.00</td>
<td>0.61</td>
<td>6.20</td>
<td>0.40</td>
<td>6.34</td>
<td>0.42</td>
</tr>
<tr>
<td>30.00</td>
<td>0.61</td>
<td>3.10</td>
<td>0.35</td>
<td>3.40</td>
<td>0.38</td>
</tr>
<tr>
<td>35.00</td>
<td>0.61</td>
<td>1.38</td>
<td>0.34</td>
<td>1.57</td>
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<td>40.00</td>
<td>0.61</td>
<td>0.51</td>
<td>0.32</td>
<td>0.62</td>
<td>0.34</td>
</tr>
</tbody>
</table>
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Compare the theoretical implied volatility to the actual observed Black volatility for each contract. Compute sum of squared errors.

Using solver in excel, find best $V$ and $\sigma_V$, subject to constraint that equity price matches. The solution is $V = 89.13$ and $\sigma_V = 0.12$. 

Models that use Option Information

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### Application of Geske: Credit Risk of Banks

<table>
<thead>
<tr>
<th>Bank Name</th>
<th>Total Assets *</th>
<th>Leverage (D/E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PNC</td>
<td>$270 billion</td>
<td>8.29</td>
</tr>
<tr>
<td>Fifth Third</td>
<td>$115 billion</td>
<td>10.08</td>
</tr>
<tr>
<td>KeyBank</td>
<td>$89 billion</td>
<td>12.45</td>
</tr>
<tr>
<td>Huntington</td>
<td>$55 billion</td>
<td>11.59</td>
</tr>
</tbody>
</table>
## Inputs of Geske Model

<table>
<thead>
<tr>
<th>Source</th>
<th>Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accounting Information</td>
<td>Short-term Liabilities (F1)</td>
</tr>
<tr>
<td></td>
<td>Long-term Liabilities (F2)</td>
</tr>
<tr>
<td></td>
<td>Maturity of Long-term Liabilities (T2)</td>
</tr>
<tr>
<td>Equity Market Information</td>
<td>Shares outstanding (E)</td>
</tr>
<tr>
<td></td>
<td>Stock price (E)</td>
</tr>
<tr>
<td></td>
<td>Equity volatility (Sigma_E)</td>
</tr>
<tr>
<td>Option Market</td>
<td>Implied option volatility (Sigma_E)</td>
</tr>
<tr>
<td>Money market</td>
<td>1-Year Treasury Constant Maturity’s yield (rf)</td>
</tr>
</tbody>
</table>
Application of Geske: Data

- Stock price
- Equity volatility \( (\sigma_E) \): Annualized standard deviation of daily logarithmic returns
- Short-term Liability (F1): Liabilities with maturity equal to or less than 1 year
  1. Non-interest bearing deposits in domestic offices
  2. Interest bearing deposits in domestic offices
  3. Non-interest bearing deposits in foreign offices
  4. Interest bearing deposits in foreign offices
  5. Federal funds purchased
  6. Repurchase agreements
  7. Commercial paper
  8. Other borrowed money with remaining maturity of 1 year or less
Application of Geske: Data

- Long-term Liability (F2): Liabilities with maturity more than 1 year
  1. Subordinated notes and debentures
  2. Trust preferred securities
- Maturity of short-term liability (T1): 1 year
- Maturity of long-term Liability (T2): Duration of Long-term Liability
- Risk free rate (Rf): Continuously compounded yield of 1-year Treasury with constant maturity
- Dividend Yield (q): Continuously compounded dividend yield
- Option Data
- Riskless 1 year CMT yield
### geske model - equity data

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Equity Value</td>
<td>64.010</td>
<td>53.220</td>
<td>70.410</td>
<td>46.630</td>
<td>28.280</td>
<td>37.590</td>
<td>47.190</td>
<td>51.380</td>
<td>58.210</td>
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<tr>
<td>Equity Volatility</td>
<td>0.131</td>
<td>0.347</td>
<td>0.469</td>
<td>0.665</td>
<td>1.041</td>
<td>1.178</td>
<td>1.140</td>
<td>1.056</td>
<td>0.619</td>
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<td>Time to Maturity of short term liability (T1)</td>
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<td>1</td>
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<tr>
<td>Time to Maturity of long term liability (T2)</td>
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<td>5</td>
<td>6</td>
<td>5</td>
<td>17</td>
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<td>Cash Dividends</td>
<td>0.630</td>
<td>0.660</td>
<td>0.660</td>
<td>0.660</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
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<tr>
<td>Riskless rate</td>
<td>0.048</td>
<td>0.024</td>
<td>0.019</td>
<td>0.005</td>
<td>0.006</td>
<td>0.005</td>
<td>0.004</td>
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<tr>
<td>Estimated Equity Price</td>
<td>64.062</td>
<td>65.216</td>
<td>70.402</td>
<td>46.632</td>
<td>28.282</td>
<td>37.590</td>
<td>47.190</td>
<td>51.380</td>
<td>58.210</td>
</tr>
<tr>
<td>Estimated Equity Volatility</td>
<td>0.151</td>
<td>0.347</td>
<td>0.469</td>
<td>0.665</td>
<td>1.041</td>
<td>1.178</td>
<td>1.140</td>
<td>1.056</td>
<td>0.619</td>
</tr>
<tr>
<td>Optimize SSE</td>
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### output

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<tbody>
<tr>
<td>Asset Value</td>
<td>335.918</td>
<td>362.352</td>
<td>376.576</td>
<td>385.322</td>
<td>521.958</td>
<td>493.189</td>
<td>482.966</td>
<td>498.977</td>
<td>457.110</td>
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<td>Asset Volatility</td>
<td>0.029159</td>
<td>0.051099</td>
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<td>0.072719</td>
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<td>0.151200</td>
<td>0.140289</td>
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<td>Short term default probability</td>
<td>0.000000</td>
<td>0.001006</td>
<td>0.003791</td>
<td>0.006999</td>
<td>0.039099</td>
<td>0.230299</td>
<td>0.330663</td>
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<td>NWC ratio</td>
<td>0.128</td>
<td>0.147</td>
<td>0.167</td>
<td>0.084</td>
<td>0.054</td>
<td>0.075</td>
<td>0.099</td>
<td>0.104</td>
<td>0.127</td>
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<tr>
<td>NRC ratio</td>
<td>0.086</td>
<td>0.082</td>
<td>0.082</td>
<td>0.097</td>
<td>0.100</td>
<td>0.105</td>
<td>0.109</td>
<td>0.114</td>
<td>0.103</td>
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</tr>
</tbody>
</table>
Default Probability 2007 to 2011

PNC

Huntington

Key

Fifth Third
Comparison of DP Under Equity Market and Option Market

PNC

Huntington

Key

Fifth Third
### Ratings from Standard & Poors For Bank Holding Companies

<table>
<thead>
<tr>
<th></th>
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<td>PNC</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A+</td>
<td>A+</td>
<td>A+</td>
<td>A-</td>
<td>A-</td>
<td>A-</td>
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<tr>
<td>Fifth Third</td>
<td>A+</td>
<td>A+</td>
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<td>A+</td>
<td>A+</td>
<td>A-</td>
<td>A-</td>
<td>BBB</td>
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<tr>
<td>Huntington</td>
<td>BBB+</td>
<td>BBB+</td>
<td>BBB+</td>
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<td>BBB+</td>
<td>BBB</td>
<td>BB</td>
<td>BB+</td>
</tr>
</tbody>
</table>

### Short-term Default Probability During Crisis

#### Bank Name
- Huntington
- Fifth Third
- KeyBank
- PNC

#### Rank as of 3Q 2009
- PNC: 1
- KeyBank: 2
- Fifth Third: 3
- Huntington: 4
CreditGrades

- In 1997, JP Morgan, with the co-sponsorship of five other institutions, established CreditGrades.
- The goal of CreditGrades, was to provide a transparent standard for quantitative credit risk.
- They wanted to establish a standardization and consistency of approach that allows the immediate application of the model to a wide array of publicly traded firms.
- The CreditGrades model is a structural model that builds on the Cox-Ross model that defaults can occur at any time.
- The Black-Cox model still produces low credit spread because assets that begin above the barrier cannot reach the barrier immediately by diffusion only.
- To increase the spreads we could incorporate jumps into the asset value process. This certainly would be helpful.
- An alternative approach would be to assume there is uncertainty in the default barrier.
- Specifically, the CreditGrades model allows the barrier itself to fluctuate randomly.
- The uncertainty in the barrier admits the possibility that the firms asset...
CreditGrades Model

\[ \frac{dV_t}{V_t} = \sigma dW_t + \mu_D dt \]
# Inputs for CreditGrades

<table>
<thead>
<tr>
<th>Input Category</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Debt Per Share</strong></td>
<td>((\text{LT debt} + \text{ST debt} + 0.5 \times (\text{other LT liability} + \text{other ST liability}) - \text{minority interest}) / # \text{of shares outstanding})</td>
</tr>
<tr>
<td><strong>Stock Price</strong></td>
<td>Equity value, stock price</td>
</tr>
<tr>
<td><strong>Equity Volatility</strong></td>
<td>Historical equity volatility, annualized standard deviation of daily equity returns over previous year</td>
</tr>
<tr>
<td><strong>Risk-free Rate</strong></td>
<td>1-year U.S. Treasury Rate</td>
</tr>
<tr>
<td><strong>Time to Maturity</strong></td>
<td>1 year</td>
</tr>
<tr>
<td><strong>Asset Specific Recovery Rate</strong></td>
<td>Assumed to be 0.6</td>
</tr>
<tr>
<td><strong>Standard Deviation of Global Mean Recovery Rate</strong></td>
<td>Assumed to be 0.4</td>
</tr>
<tr>
<td><strong>Global Recovery Rate</strong></td>
<td>Assumed to be 0.5</td>
</tr>
<tr>
<td><strong>Reference Stock Price</strong></td>
<td>Assume it is the company's stock price</td>
</tr>
</tbody>
</table>
Default will not occur if the asset value never falls below the barrier before date $t$. 

The survival function can be computed analytically and is given by $PS(t) = P(A_s > D_s, \forall s < t) = P(X(s) > \ln D_0 A_0 - \lambda^2, \forall s < t) = N_2(\lambda^2 + \ln(d\lambda), \lambda G_t^2 + \ln(d\lambda); -\lambda G_t^2 - \ln(d\lambda); -\lambda G_t^2)$, where $G_t^2 = \sigma^2 V_t + \lambda^2 d = V_0(0) D_0 e^{\lambda^2}$ and $N_2(x, y; \rho)$ is the cumulative standard bivariate normal distribution, with correlation $\rho$.

With the risk neutral survival probability in hand we can compute the price of bonds given recovery rate assumptions and cds rates. For example, if the recovery rate upon default is zero then the price of a zero coupon bond is just $B_0 = P(0, T) PS(T)$. 

*CreditGrades*
Default will not occur if the asset value never falls below the barrier before date $t$.

The survival function can be computed analytically and is given by $PS(t)$ where

$$PS(t) = P(A_s > D_s, \text{ for all } s < t) = P(X(s) > \ln D_0 A_0 - \lambda^2, \text{ for all } s < t)$$

$$= N_2\left(\frac{\lambda}{2} + \frac{\ln(d)}{\lambda}, -\frac{G_t}{2} + \frac{\ln(d)}{\lambda}; \frac{\lambda}{G_t}\right) - dN_2\left(\frac{\lambda}{2} + \frac{\ln(d)}{\lambda}, -\frac{G_t}{2} - \frac{\ln(d)}{\lambda}; \frac{\lambda}{G_t}\right)$$

where

$$G_t^2 = \sigma_V^2 t + \lambda^2$$

$$d = \frac{V(0)}{D(0)} e^{\lambda^2}$$

and $N_2(x, y; \rho)$ is the cumulative standard bivariate normal distribution, with correlation $\rho$.  


Default will not occur if the asset value never falls below the barrier before date t.

The survival function can be computed analytically and is given by \( PS(t) \) where

\[
PS(t) = P(A_s > D_s, \text{ for all } s < t) = P(X(s) > \ln D_0 A_0 - \lambda^2, \text{ for all } s < t)
\]

\[
= N_2\left(\frac{\lambda}{2} + \frac{\ln(d)}{\lambda}, -\frac{G_t}{2} + \frac{\ln(d)}{G_t}; \frac{\lambda}{G_t}\right) - dN_2\left(\frac{\lambda}{2} + \frac{\ln(d)}{\lambda}, -\frac{G_t}{2} - \frac{\ln(d)}{\lambda}; \frac{\lambda}{G_t}\right)
\]

where

\[
G_t^2 = \sigma^2 V_t t + \lambda^2
\]

\[
d = \frac{V(0)}{D(0)} e^{\lambda^2}
\]

and \( N_2(x, y; \rho) \) is the cumulative standard bivariate normal distribution, with correlation \( \rho \).

With the risk neutral survival probability in hand we can compute the price of bonds given recovery rate assumptions and cds rates.
CreditGrades

Default will not occur if the asset value never falls below the barrier before date t.

The survival function can be computed analytically and is given by $PS(t)$ where

$$PS(t) = P(A_s > D_s, \text{ for all } s < t) = P(X(s) > \ln D_0 A_0 - \lambda^2, \text{ for all } s < t)$$

$$= N_2(\frac{\lambda}{2} + \frac{\ln(d)}{\lambda}, -\frac{G_t}{2} + \frac{\ln(d)}{\lambda} ; \frac{\lambda}{G_t}) - dN_2(\frac{\lambda}{2} + \frac{\ln(d)}{\lambda}, -\frac{G_t}{2} - \frac{\ln(d)}{\lambda};$$

where

$$G_t^2 = \sigma_V^2 t + \lambda^2$$

$$d = \frac{V(0)}{D(0)} e^{\lambda^2}$$

and $N_2(x, y; \rho)$ is the cumulative standard bivariate normal distribution, with correlation $\rho$.

With the risk neutral survival probability in hand we can compute the price of bonds given recovery rate assumptions and cds rates.

For example, if the recovery rate upon default is zero then the price of a zero coupon bond is just

$$B_0 = P(0, T) PS(T).$$
Case Study: Comparing a Few Models

- Collected publicly available information for 160 companies sample over 5 years after IPO
- Analyzed data and estimated risk neutral probability of default using 3 structural models: Merton, Delianedis and Geske, and CreditGrades
- Compared default prediction of 3 models with basic statistics, ROC curve and K-S value and regression analysis.
- The firms with IPO offer date occurred during 1996 to 2004 period.
- 80 firms were delisted for poor performance in 5 years after IPO
- 80 firms survived the first 5 years.
Case Study – Firearms Training Systems Inc.

Company Profile:
Firearms Training Systems Inc. produces interactive simulation systems for weapons training. The Company offers training systems including the handling and use of small and supporting arms as well as hazardous situations simulation and solutions for military, law enforcement agencies, emergency agencies, and sporting goods stores.

- Founded in 1984
- Based in Suwanee, Georgia
- Going Public in 1996
- Delisted in 2000
Case Study

Case Study – Firearms Training Systems Inc.

Stock Price through IPO Date to Delisted Date

[Graph showing stock price fluctuations over time]

Year after IPO

[Another graph showing percentage change over time]
Case Study – LANDEC Corporation

Company Profile:
Landec Corporation designs, develops, manufactures, and sells polymer products for food and agricultural products, medical devices, and licensed partner applications, which incorporate its patented polymer technologies. Landec Corporation sells its products in the United States, Indonesia, Taiwan, Canada, Belgium, Japan, and internationally.

- Founded in 1986
- Based in Menlo Park, California
- Going public in 1996
Case Study – LANDEC Corporation

Adjusted Close Price for 5 years since IPO

Merton
GeskeTotal
GeskeShortTerm
CreditGrades
Comparing 3 Models: Default Firms Boxplot

PD as computed for each time horizon and each model (Default Firms)
Comparing 3 Models: Surviving Firms Boxplot

PD as computed for each horizon and each model (Survival Firms)
How to use these structural models

- What is the option market view
- What is the CDS view
- Links between CDS and Options
- Difference in model predictions
- Use of models with jumps
- Relative Rankings
- Obvious Extensions
- Trinomial Models
What is to follow

- Deeper dive into Capital Structure
- Role of Taxes, Bankruptcy Costs
- Agency costs
- Investment Opportunities and Growth Options
- Dynamic Capital Structure Issues
- Other topics, reorganizations, cash and liquidity......