Structured MSc in Mathematical Sciences - August 2016 intake

Review Course Abstracts

7-25 November

Algebra
Karin-Therese Howell (Stellenbosch)

The course will have an Algebra and a Linear Algebra component. Under Algebra we will discuss functions, relations, partitions, groups, morphisms, quotients, the Isomorphism Theorems for groups and direct products. As part of the Linear Algebra component we will discuss linear spaces, linear functionals and operators, matrices, change of basis, eigenvalues and eigenvectors and the normal form.

Differential Equations
Patrick Dorey (Durham)

Recognising different pdes: independent/dependent variables; order of a pde; linear - semilinear - quasilinear - fully nonlinear pdes. Quick revision of odes: separable, 1st order linear, 2nd order constant coefficient odes Picard-Lindelof theorem. Well-posed pdes: existence, uniqueness, continuous dependence on initial/boundary data and parameters. First-order linear pdes, method of characteristics First-order quasilinear pdes. Conservation laws and kinematic waves. Solutions with discontinuities; rarefaction waves. Application: traffic flow a la Lighthill and Whitham Linear and non-linear wave equations, possible discussion of either exactly-solvable examples or of collective coordinate approximations, perhaps combined with Python programs and issues with numerics (instability).

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28 November - 16 December

Model Theory
Gareth Boxall (Stellenbosch)

Model theory is a branch of mathematical logic which has applications in several other areas of mathematics including algebraic geometry, number theory and combinatorics. This course will introduce model theory from the beginning, including some preliminaries on ordinals and cardinals. The focus will then be on reviewing several interesting applications. In particular, we shall devote significant time to non-standard analysis.

Data Analytics
Pete Grindrod (Oxford)

This course provides an introduction to the recent but influential subject of Big Data and Data Analytics. After setting the scene, the basic analytic techniques are introduced. Many examples are discussed to demonstrate the benefits of this approach to modelling big data sets; use of tools is also included. These methods and tools are applied to practical problems, to the extent that one benefit of the course will be an appreciation of the entrepreneurial consequences of this approach.

Reference: P. Grindrod, “Mathematical Underpinnings of Analytics”, Oxford University Press.

9-27 January 2017

Rafael Nepomechie (Miami)
Quantum Mechanics and Quantum Spin Chains

The first week will be devoted to reviewing elements of classical mechanics (Lagrangian, Hamiltonian, and Poisson brackets) and linear algebra using Dirac bra-ket notation. The eigenvalue problem and simultaneous diagonalization will be emphasized. In the second week, the principles of quantum mechanics will be introduced, using the free particle, the harmonic oscillator and the two-state (spin-1/2) system as the working examples. Students will then learn about higher (s > 1/2) spin, which provides an ideal introduction to Lie algebras and their irreducible representations. Using the notion of tensor product to describe a composite system of two spins, students will be introduced to the Clebsch-Gordan theorem. Students will gain valuable "hands on" experience by using Sage to explicitly obtain both irreducible and reducible representations of the SU (2) generators. The capstone of this course, making non-trivial use of all of these concepts, will be an introduction to the Heisenberg quantum spin chain (a ring of N spin-1/2 spins with nearest-neighbor isotropic interactions), which is of fundamental importance in theoretical physics. Students will first study this model numerically for small values of N. They will classify the states according to energy, momentum and spin, and understand degeneracies with the help of the Clebsch-Gordan theorem. Finally, they will reproduce these results from the Bethe ansatz, and go further by computing the ground-state properties in the infinite-N limit. Additional mathematical and computer techniques (including the Newton-Raphson method, recursive programming, and Fourier transforms) will also be reviewed and used.

Fluid Dynamics
Grae Worster (Cambridge) and Richard Katz (Oxford)

Fluids are all around us, from the air we breath to the oceans that determine our climate and from oil that powers our industries to metals that are cast into machinery. The study of fluid dynamics requires sophisticated applications of mathematics and the ability to translate physical problems into mathematical language and back again. The course begins by building a fundamental understanding of viscous fluid flows in the context of unidirectional flows. In more general, higher dimensional flows, pressure gradients are generated within a fluid to deflect the flow around obstacles rather than the fluid being compressed in front of them, and an understanding of the coupling between momentum and mass conservation through the pressure field is key to the understanding and analysis of fluid motions. We will use simple experiments to illustrate and motivate our mathematical understanding of fluid flow. Prerequisite for the course is fluency with differential equations and vector calculus. No previous knowledge of fluid dynamics will be assumed.

Introduction to Random Systems, Information Theory and Related Topics
Stephane Ouvry (Paris-Sud University)

This course is an introduction to various random systems, probability theory, Shannon information entropy and some related topics, with a special emphasis on their mathematical aspects. In particular students will learn:
• Probability calculus and the central limit theorem
• Application to Monte Carlo sampling

- Application to random walks on a line
- Random walks on a square lattice and their algebraic area
- Random permutations and their application to
  - the statistical “curse” problem in sailing boat regattas
  - the Diaconis shuffling cards trick
- Shannon statistical entropy and information theory
- LZW compression algorithm
- **Application to the identification of languages**

11-14 and 17-20 January 2016
Mathematics in Industry Study Group

The Mathematics in Industry Study Group is a five-day workshop at which academic researchers and graduate students work collaboratively with representatives from industry on research problems submitted by local industry. (See https://www.aims.ac.za/en/research-centre/workshops-conferences/past/mathematics-in-industry-study-group-misgsa-2015 for details of the 2015 workshop).

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30 January to 17 February 2016

Analytical Techniques in Mathematical Biology
Wilson Lamb (Strathclyde)
Mathematical models arising in the natural sciences often involve equations which describe how the phenomena under investigation evolve in time. When time is regarded as a continuous variable such evolution equations usually take the form of differential equations. In this course a number of mathematical techniques will be presented for analysing a range of evolution equations that arise in Biomathematics, particularly in population dynamics. The emphasis will be placed on determining qualitative features of solutions, such as the long-term behaviour. Different types of equations will be examined, but a unifying theme will be provided by developing methods from a dynamical systems point of view and using some results from functional analysis. To fix ideas, the course will begin with some simple one-dimensional models from population dynamics, such as the Malthusian and Verhulst equations. Structured population models arising in epidemiology, such as the SIS and SIR models, and multispecies models, such as the Lotka –Volterra predator-prey equations, will be considered next. The latter models result in non-linear systems of ordinary differential equations and their analysis involves higher (but still finite) dimensional dynamical systems theory. To give an indication of the need, in some problems, to work within an infinite-dimensional setting, we shall conclude by examining the notion of diffusion-driven (or Turing) instability in reaction-diffusion type partial differential equations and discuss mathematical models of pattern formation (e.g. in animal coats) that involve such equations.

Groups and Geometry
Alan Beardon (Cambridge)

Starting with ideas about groups of transformations, we investigate groups of isometries of Euclidean, spherical and hyperbolic geometries. We end with a discussion of topological groups, and groups of matrices, in which the group elements themselves move continuously within the group, and we discuss the geometry associated with these motions.

Douw Steyn (British Columbia) and Stefano Galmarinia (Institure for Environment and Sustainability, Italy)
This course will be based on the AIMS Library Series book of the same title. Book content will be covered in lectures and/or seminars, and will be augmented by a series of analytical/numerical exercises designed to illustrate and give practice in the course material.

27 February to 17 March

Entrepreneurship case studies.
Stefan Jaehnichen (TU Berlin)

Computational Algebra
Wolfram Decker and Gerhard Pfister, Kaiserslautern

Groebner bases and Buchberger's algorithm for ideals and modules will be studied. Applications to commutative algebra, selected problems in singularity theory and algebraic geometry will be given as well as applications to electronics and engineering. The course includes an introduction to the computer algebra system SINGULAR and its programming language.

Bioinformatics
Paolo Zuliani (Newcastle)

We present some of the most used mathematical and computational techniques for modelling biological systems, with a focus on genetic and biochemical networks. We introduce metabolic modelling using flux balance analysis, and signalling pathway modelling using ODEs (Ordinary Differential Equations) and stochastic discrete models (Gillespie's algorithm). For biological sequence analysis we introduce hidden Markov models and related algorithms (e.g., expectation-maximisation). Finally, we present very recent work on parameter synthesis for ODE models, using SMT (Satisfiability Modulo Theory) techniques. This course does NOT assume any Biology background.

Experimental Topology: from field theory to information science
Jeff Murugan (Cape Town)

Topology, unlike geometry, is about the global properties of objects, properties that are preserved through deformations, twistings, and stretchings (but not tearing). For example, a soccer ball is topologically the same as a rugby ball (into which it can be deformed by stretching) but no amount of twisting or stretching will deform the soccer ball into a donut! The property of the torus that differentiates it from the sphere is called its genus. The genus of an object counts the number of handles that the object has, hence the joke that a topologist is someone who can't tell her coffee cup from her donut. It is an example of a topological invariant, numbers which do not change when the object is deformed and which characterise its global properties. In recent years, these powerful topological methods, once the domain of only pure mathematics, have made their way into a spectrum of applied mathematical research, from network theory, to data analysis, to quantum field theory. This course is an introduction to some of these methods and their application in mathematical physics.

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20 March to 7 April

Introduction to Quantum Field Theory
Robert de Mello Koch (Witwatersrand)

Quantum Field Theory provides the successful unification of quantum mechanics and special relativity. The course begins with a description of classical field theory, using classical electrodynamics as a concrete example. In particular, we discuss both the Lagrangian and Hamiltonian versions of the dynamics and we derive Noether's theorem. The need for a description employing fields in any theory consistent with Special Relativity is developed. The real and complex scalar fields are dealt with in the homework problems.

The quantum field theory of the real scalar field is then developed, using canonical quantization. The spectrum of the theory is obtained and time ordered correlation functions are introduced. Using the interactionpicture, the Feynman rules for interacting quantum field obtained. If time permits we will recover these results using a path integral approach.

The course aims to provide a solid conceptual framework. For example, we explicitly describe how a simple two point function computation leads to the Yukawa force as an exchange of mesons. We also explain why much of the structure of the theory follows directly from the structure of quantum mechanics and special relativity.

The Numerical Solution of Differential Equations
Lyonell Boulton Febres (Herriot-Watt)

The aim of this course will be to provide an overview of the basic techniques, tools and theory for solving both ordinary differential equations (ODE) and partial differential equations (PDE) numerically. For the ODE part of the course, we will focus our attention on the theory of numerical solution of evolution problems, and its many applications. Simple examples will be discussed, including diffusion of heat in a bar. More complex examples, such as the transport and diffusion of chemicals (e.g oil) through an underground reservoir, will also be considered. For the PDE part of the course, we will focus on the finite element method in two and three dimensions. We will include applications from spectral theory, the theory of sound and elementary models from quantum mechanics. During this course, the students will be presented with various numerical algorithms. All these algorithms will be made available in Sage, Octave and FreeFem++. I am aware that some of the students will not be familiar with the latter. As part of the course, I will include a crash course on the elements of programing in FreeFem++.

Algebraic biology

Matt Macauley (Clemson)

Mathematical biology has been transformed over the past 15 years by researchers using novel tools from discrete math and computational algebra to tackle old and new problems. For example, many systems such as gene regulatory networks have been traditionally modeled using differential equations. However, a new popular trend is to use finite dynamical systems such as Boolean networks. In this setting, the local functions and the dynamical system map can be expressed as multivariable polynomials. This opens the door to using the powerful toolbox of computational algebra to attack classic problems in systems biology. In this class, students will be introduced to this new and exciting field known as "algebraic mathematical biology."
